Cube Testers and Key-Recovery Attacks on Reduced-Round MD6 and Trivium

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Cube attacks

Timeline

Aug 08: Shamir presents cube attacks at CRYPTO

Sep 08: Dinur/Shamir paper on ePrint,
attack on 771-round Trivium

Oct 08: cube attacks reported on 14-round MD6

Oct 08: cube testers reported on 18-round MD6

Dec 08: Dinur/Shamir paper accepted to EUROCRYPT

Jan 09: cube testers reported on Shabal

Cube attacks in a nutshell

Can attack any primitive with **secret and public variables**

- keyed hash functions
- stream ciphers
- block ciphers
- MACs

Target algorithms with **low-degree** components

- stream ciphers based on low-degree NFSR
- hash functions with only XORs and a few ANDs

Cube attacks in a nutshell

Requirements of the attacker:

- only **black-box access** to the function
- negligible memory

Cube attacks work in 2 phases

- **precomputation**: chosen keys and chosen IVs
- **online**: fixed unknown key and chosen IVs

Key observation 1

Any function

$$f : \{0,1\}^m \mapsto \{0,1\}^n$$

admits an **algebraic normal form** (ANF)

Example: $f : \{0,1\}^{10} \mapsto \{0,1\}^4$

$$\begin{aligned}
f_1(x) &= x_1x_2 + x_2x_8x_9 + x_3x_4x_5x_6x_7 \\
f_2(x) &= x_2x_4 + x_6x_8x_9 + x_5x_6x_7x_8x_9x_{10} \\
f_3(x) &= 1 \\
f_4(x) &= 1 + x_1 + x_3 + x_5
\end{aligned}$$

Key observation 2

Computation of the largest monomial's coefficient

$$\begin{aligned} f(x_1, x_2, x_3, x_4) &= x_1 + x_3 + x_1x_2x_3 + x_1x_2x_4 \\ &= x_1 + x_3 + x_1x_2x_3 + x_1x_2x_4 + 0 \times x_1x_2x_3x_4 \end{aligned}$$

Sum over all values of (x_1, x_2, x_3, x_4):

$$f(0,0,0,0) + f(0,0,0,1) + f(0,0,1,0) + \cdots + f(1,1,1,1) = 0$$

Key observation 3

Evaluation of factor polynomials

$$\begin{aligned} f(x_1, x_2, x_3, x_4) &= x_1 + x_3 + x_1 x_2 x_3 + x_1 x_2 x_4 \\ &= x_1 + x_3 + x_1 x_2 (x_3 + x_4) \end{aligned}$$

Fix x_3 and x_4, sum over all values of (x_1, x_2):

$$\begin{aligned} \sum_{(x_1,x_2)\in\{0,1\}^2} f(x_1, x_2, x_3, x_4) &= 4 \times x_1 + 4 \times x_3 + 1 \times (x_3 + x_4) \\ &= x_3 + x_4 \end{aligned}$$

Key observation 3

Evaluation of factor polynomials

$$f(x_1, x_2, x_3, x_4) \quad = \quad \cdots + x_1 x_2 (x_3 + x_4)$$

Fix x_3 and x_4, sum over all values of (x_1, x_2):

$$\sum_{(x_1,x_2)\in\{0,1\}^2} f(x_1, x_2, x_3, x_4) \quad = \quad x_3 + x_4$$

Terminology

$$f(x_1, x_2, x_3, x_4) = x_1 + x_3 + x_1 x_2 (x_3 + x_4)$$

$(x_3 + x_4)$ is called the **superpoly** of the **cube** $x_1 x_2$

Evaluation of a superpoly

x_3 and x_4 fixed and unknown

$f(\cdot, \cdot, x_3, x_4)$ queried as a **black box**

ANF unknown, except: $x_1 x_2$'s superpoly is $(x_3 + x_4)$

$$f(x_1, x_2, x_3, x_4) = \cdots + x_1 x_2 (x_3 + x_4) + \cdots$$

Query f to evaluate the superpoly:

$$\sum_{(x_1, x_2) \in \{0,1\}^2} f(x_1, x_2, x_3, x_4) = x_3 + x_4$$

Key-recovery attack

On a stream cipher with key k and IV v

$$f : (k, v) \mapsto \text{first keystream bit}$$

Offline: find cubes with linear superpolys

$$\begin{aligned}
f(k, v) &= \cdots + v_1 v_3 v_5 v_7 (k_2 + k_3 + k_5) + \cdots \\
f(k, v) &= \cdots + v_1 v_2 v_6 v_8 v_{12} (k_1 + k_2) + \cdots \\
\cdots &= \cdots \\
f(k, v) &= \cdots + v_3 v_4 v_5 v_6 (k_3 + k_4 + k_5) + \cdots
\end{aligned}$$

(reconstruct the superpolys with linearity tests)

Online: evaluate the superpolys, solve the system

Cube testers

Cube testers in a nutshell

Like cube attacks:

- need only black-box access
- target primitives with secret and public variables and
- built on low-degree components

Unlike cube attacks:

- give **distinguishers** rather than key-recovery
- don't require low-degree functions
- need **no precomputation**

Basic idea

Detect structure (nonrandomness) in the superpoly,

using **algebraic property testers**

A tester for property $\mathcal{P}$ on the function f:

- makes (adaptive) queries to f
- accepts when f satisfies $\mathcal{P}$
- rejects with bounded probability otherwise

Examples of efficiently testable properties

- balance
- linearity
- low-degree
- constantness
- presence of linear variables
- presence of neutral variables

General characterization by Kaufman/Sudan, *STOC' 08*

Superpolys attackable by testing...

... **low-degree** (6)

$$\cdots + x_1 x_2 x_3 (x_5 x_6 + x_7 x_{21} + x_6 x_9 x_{20} x_{30} x_{40} x_{50}) + \cdots$$

... **neutral variables** (x_6)

$$\cdots + x_1 x_2 x_3 x_4 x_5 \cdot g(x_7, x_8, \ldots, x_{80}) + \cdots$$

... **linear variables** (x_6)

$$\cdots + x_1 x_2 x_3 x_4 x_5 \cdot (x_6 + g(x_7, x_8, \ldots, x_{80})) + \cdots$$

Results

MD6

Presented by Rivest at CRYPTO 2008

Submitted to the SHA-3 competition

- quadtree structure
- construction RO-indifferentiable
- low-degree compression function
- at least **80 rounds**
- best attack by the designers: 12 rounds

MD6's compression function

$$\{0,1\}^{64\times 89} \mapsto \{0,1\}^{64\times 16}$$

Input: 64-bit words $A_0.A_1,\ldots,A_{88}$

Compute the A_i's with the recursion

$$x \leftarrow S_i \oplus A_{i-17} \oplus A_{i-89} \oplus (A_{i-18} \wedge A_{i-21}) \oplus (A_{i-31} \wedge A_{i-67})$$
$$x \leftarrow x \oplus (x \gg r_i)$$
$$A_i \leftarrow x \oplus (x \ll \ell_i)$$

- round-dependent constant S_i
- quadratic step, at least 1280 steps

Results on MD6

Cube attack (key recovery)

- on the **14-round** compression function
- recover any 128-bit key
- in time $\approx 2^{22}$

Cube testers (testing balance)

- detect nonrandomness on **18 rounds**
- detect nonrandomness on **66 rounds** when $S_i = 0$
- in time $\approx 2^{17}$, 2^{24}, resp.

Trivium

Stream cipher by De Cannière and Preneel, 2005

eSTREAM HW portfolio

- 80-bit key and IV
- 3 quadratic NFSRs
- 1152 initialization rounds
- best attack on 771 rounds (cube attack)

Cube testers on Trivium

Test the presence of **neutral variables**

Distinguishers (only choose IVs)

- 2^{24}: 772 rounds
- 2^{30}: 790 rounds

Nonrandomness (assumes some control of the key)

- 2^{24}: 842 rounds
- 2^{27}: 885 rounds

Full version: 1152 rounds

Conclusions

Cube testers

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- more general than classical cube attacks
- no precomputation
- “polymorphic”

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- only gives distinguishers
- only finds feasible attacks
- relevant for a minority of functions (like cube attacks)

Open issues

How to predict the existence of unexpected properties?

How to find the best cubes?

Attack on (reduced versions of) other algorithms:

Grain, ESSENCE, Keccak, Luffa, Shabal,…

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